

# $\rho$ -meson form factors and QCD sum rules.

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## Abstract

We present predictions for  $\rho$ -meson form factors obtained from the analysis of QCD sum rules in next-to-leading order of perturbation theory. The radiative corrections turn out to be sizeable and should be taken into account in rigorous theoretical analysis.

## 1 Introduction

The method of QCD sum rules [1] is designed to estimate low-energy characteristics of hadrons, such as masses, decay constants and form factors. Within this framework we analyze the correlation function of currents in deep euclidean region with the help of operator product expansion, which allows us to take into account both perturbative and nonperturbative contributions. The presence of latter could be traced to the non-vanishing values of vacuum QCD condensates. Physical quantities, we are interested in, are determined by matching this correlator to its phenomenological representation.

In this work we performed an analysis of three-point sum rules for  $\rho$ -meson form factors at intermediate momentum transfer. Basically, it is an extension of already available LO analysis [2] to include radiative corrections. To compute radiative corrections we used the technic already developed and tested in the analysis of pion electromagnetic form factor within NLO QCD sum rule setup both with pseudoscalar and axial-vector pion interpolating currents [3, 4].

The paper is organized as follows. In section 2 we describe our framework and give explicit expressions for next-to-leading order corrections to double spectral density. Section 3 contains our numerical analysis and expressions for the contributions of gluon and quark condensates. Finally, in section 4 we draw our conclusions.

## 2 The method

To determine  $\rho$ -meson electromagnetic form factors we will use the method of three-point QCD sum rules. Within this framework  $\rho$ -meson is described as a result of an action of vector interpolating current on vacuum state. We define the vacuum to  $\rho$ -meson transition matrix element of vector current as

$$\langle 0 | j_\mu | \rho^+, \epsilon \rangle = \frac{m_\rho^2}{g_\rho} \epsilon_\mu, \quad (1)$$

where  $m_\rho$  is the  $\rho$ -meson mass,  $g_\rho$  is the  $\rho - \gamma$  coupling constant ( $g_\rho^2/4\pi = 1.27$ ) and  $\epsilon_\mu$  stands for  $\rho$ -meson polarization vector. Next, assuming parity and time-reversal invariance, the general

expression for  $\rho$ -meson electromagnetic vertex could be written in terms of three form factors:

$$\langle \rho^+(p', \epsilon') | j_\mu^{\text{el}} | \rho^+(p, \epsilon) \rangle = -\epsilon_\beta^* \epsilon_\alpha \left\{ [(p' + p)_\mu g_{\alpha\beta} - p'_\alpha g_{\beta\mu} - p_\beta g_{\alpha\mu}] F_1(Q^2) + [g_{\mu\alpha} q_\beta - g_{\mu\beta} q_\alpha] F_2(Q^2) + \frac{1}{m_\rho^2} p'_\alpha p_\beta (p + p')_\mu F_3(Q^2) \right\}, \quad (2)$$

where  $j_\mu^{\text{el}} = e_u \bar{u} \gamma_\mu u + e_d \bar{d} \gamma_\mu d$ , the momenta of initial and final state  $\rho$ -mesons were denoted by  $p$ ,  $p'$  and  $Q^2 = -q^2$  ( $q = p - p'$ ) is the square of momentum transfer.  $F_1$ ,  $F_2$  and  $F_3$  are electric, magnetic and quadrupole form factors. At zero momentum transfer these form factors are expressed through the usual static quantities of  $\rho$ -meson charge, magnetic moment ( $\mu$ ) and quadrupole moment ( $D$ ):

$$F_1(0) = 1, \quad (3)$$

$$F_2(0) = \mu - 1, \quad (4)$$

$$F_3(0) = \frac{1}{2}(\mu - 1) - \frac{m_\rho^2}{4} D. \quad (5)$$

Within approach of QCD sum rules the theoretical estimates of  $\rho$ -meson form factors follow from the analysis of the following three-point correlation function:

$$\Pi_{\mu\alpha\beta} = i^2 \int d^4x d^4y e^{i(p' \cdot x - p \cdot y)} \langle 0 | T\{j_\beta^+(x), j_\mu^{\text{el}}(0), j_\alpha(y)\} | 0 \rangle, \quad (6)$$

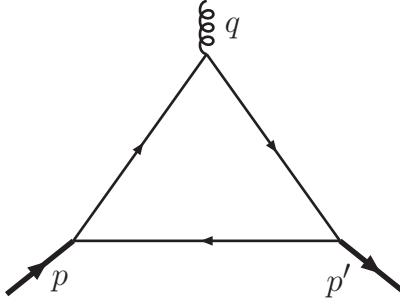


Figure 1: LO diagram

The scalar amplitudes  $\Pi_i$  in front of different Lorentz structures are the functions of kinematical invariants, i.e.  $\Pi_i = \Pi_i(p^2, p'^2, q^2)$ . In the region of large euclidean momenta  $p^2, p'^2, q^2 < 0$  this correlation function could be studied with the use of ordinary perturbative QCD. The calculation of QCD expression for three-point correlator is done through the use of operator product expansion (OPE) for the T-ordered product of currents. As a result of OPE one obtains besides leading perturbative contribution also power corrections, given by vacuum QCD condensates. We will return to the discussion of power correction to QCD sum rules later after the definition of Borel transform for our correlation function. Now let us discuss the perturbative contribution. The calculation of perturbative contribution could be conveniently performed with the use of double dispersion representation in variables  $s_1 = p^2$  and  $s_2 = p'^2$  at  $q^2 < 0$ :

$$\Pi_{\mu\alpha\beta}^{\text{pert}}(p^2, p'^2, q^2) = \frac{1}{(2\pi)^2} \int \frac{\rho_{\mu\alpha\beta}^{\text{pert}}(s_1, s_2, Q^2)}{(s_1 - p^2)(s_2 - p'^2)} ds_1 ds_2 + \text{subtractions} \quad (7)$$

The integration region in (7) is determined by condition <sup>1</sup>

$$-1 \leq \frac{s_2 - s_1 - q^2}{\lambda^{1/2}(s_1, s_2, q^2)} \leq 1 \quad (8)$$

and

$$\lambda(x_1, x_2, x_3) = (x_1 + x_2 - x_3)^2 - 4x_1x_2. \quad (9)$$

The double spectral density  $\rho_{\mu\alpha\beta}^{\text{pert}}(s_1, s_2, Q^2)$  is searched in the form of expansion in strong coupling constant:

$$\rho_{\mu\alpha\beta}^{\text{pert}}(s_1, s_2, Q^2) = \rho_{\mu\alpha\beta}^{(0)}(s_1, s_2, Q^2) + \left(\frac{\alpha_s}{4\pi}\right) \rho_{\mu\alpha\beta}^{(1)}(s_1, s_2, Q^2) + \dots \quad (10)$$

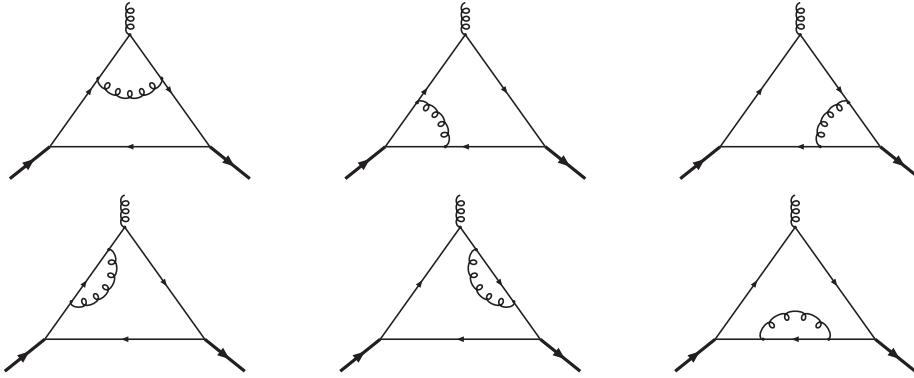


Figure 2: NLO diagrams

At leading order in coupling constant we have only one diagram depicted in Fig. 1, contributing to three-point correlation function. At next to leading order we have 6 diagrams shown in Fig. 2. The calculation of corresponding double spectral density was performed with the standard use of Cutkosky rules. In the kinematic region  $q^2 < 0$ , we are interested in, there is no problem in applying Cutkosky rules for determination of  $\rho_{\mu\alpha\beta}(s_1, s_2, Q^2)$  and integration limits in  $s_1$  and  $s_2$ . The non-Landau type singularities, not accounted for by Cutkosky prescription, do not show up here. The calculation could be considerably simplified with the use of Lorentz decomposition of double spectral density based on a fact, that our spectral density is subject to three transversality conditions:  $\rho_{\mu\alpha\beta}q_\mu = \rho_{\mu\alpha\beta}p_\alpha = \rho_{\mu\alpha\beta}p'_\beta = 0$

$$\begin{aligned} \rho^{\mu\alpha\beta} = & A_1[(Q^2 + x)p_1^\alpha - (x + y)p_2^\alpha][(y - x)p_1^\beta + (Q^2 + x)p_2^\beta][(Q^2 + y)p_1^\mu + (Q^2 - y)p_2^\mu] \\ & - \frac{1}{2}A_2[(Q^2 + y)p_1^\mu + (Q^2 - y)p_2^\mu][(Q^2 + x)g^{\alpha\beta} - 2p_1^\beta p_2^\alpha] \\ & - \frac{1}{2}A_3[(Q^2 + x)p_1^\alpha - (x + y)p_2^\alpha][2(p_2^\beta - p_1^\beta)p_2^\mu + (Q^2 + y)g^{\mu\beta}] \\ & - \frac{1}{2}A_4[(x - y)p_1^\beta - (Q^2 + x)p_2^\beta][2(p_2^\alpha - p_1^\alpha)p_1^\mu + (y - Q^2)g^{\mu\alpha}], \end{aligned} \quad (11)$$

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<sup>1</sup>In our case this inequality is satisfied identically.

where  $x = s_1 + s_2$ ,  $y = s_1 - s_2$  and  $p_1 = p$ ,  $p_2 = p'$ . The four independent structures  $A_i$  (we suppressed the dependence on kinematical invariants) are given by a solution of system of linear equations:

$$I_1 = \rho_{\mu\alpha\beta} p_1^\mu p_2^\alpha p_1^\beta = \frac{k^2}{8} (kA_1 - A_2 - A_3 - A_4) \quad (12)$$

$$I_2 = \rho_{\mu\alpha\beta} p_1^\mu g^{\alpha\beta} = \frac{k}{4} (x + Q^2) (kA_1 - 3A_2 - A_3 - A_4) \quad (13)$$

$$I_3 = \rho_{\mu\alpha\beta} p_2^\alpha g^{\mu\beta} = \frac{k}{4} (y + Q^2) (kA_1 - A_2 - 3A_3 - A_4) \quad (14)$$

$$I_4 = \rho_{\mu\alpha\beta} p_1^\beta g^{\mu\alpha} = -\frac{k}{4} (y - Q^2) (kA_1 - A_2 - A_3 - 3A_4), \quad (15)$$

where  $k = \lambda(s_1, s_2, -Q^2)$ . Solving this system it is easy to find explicit expressions for  $A_i$  in terms of  $I_i$  (functional dependence on kinematical invariants is assumed):

$$A_1 = \frac{20}{k^3} I_1 - \frac{2}{(Q^2 + x)k^2} I_2 - \frac{2}{(Q^2 + y)k^2} I_3 - \frac{2}{(Q^2 - y)k^2} I_4, \quad (16)$$

$$A_2 = \frac{4}{k^2} I_1 - \frac{2}{(Q^2 + x)k} I_2, \quad (17)$$

$$A_3 = \frac{4}{k^2} I_1 - \frac{2}{(Q^2 + y)k} I_3, \quad (18)$$

$$A_4 = \frac{4}{k^2} I_1 - \frac{2}{(Q^2 - y)k} I_4. \quad (19)$$

At Born level and expressions for  $I_i$  are easy to find and they are given by ( $s_3 = Q^2$ ) [2]:

$$I_1^{(0)} = -\frac{3s_1 s_2 s_3}{2k^{1/2}}, \quad (20)$$

$$I_2^{(0)} = -\frac{3s_1 s_2}{k^{1/2}}, \quad (21)$$

$$I_3^{(0)} = -\frac{3s_2 s_3}{k^{1/2}}, \quad (22)$$

$$I_4^{(0)} = -\frac{3s_1 s_3}{k^{1/2}}, \quad (23)$$

The calculation of NLO radiative corrections to double spectral density is in principle straightforward. One just needs to consider all possible double cuts of diagrams, shown in Fig. 2. However, the presence of collinear and soft infrared divergences calls for appropriate regularization of arising divergences at intermediate steps of calculation and makes the whole analytical calculation quite involved. We will present the details of NLO calculation in one of our future publications. Here we give only final results:

$$\begin{aligned} k^{1/2} I_1^{(1)} &= -s_1^3 + s_2 s_1^2 + s_2^2 s_1 - s_2^3 + (s_1 + s_2) s_3^2 - s_1^2 s_3 - s_2^2 s_3 + s_1 s_2 s_3 \Big[ \\ &\quad -16 \log^2(v_1) - 16 \log(v_3) \log(v_1) - 16 \log(v_4) \log(v_1) \\ &\quad + 2 \log(v_1) - 4 \log^2(v_3) - 4 \log^2(v_4) - 2 \log(v_2) - 2 \log(v_3) - 8 \log(v_3) \log(v_4) \\ &\quad - 8 \text{Li}_2\left(\frac{x_2}{x_1}\right) - 8 \text{Li}_2\left(\frac{y_1}{y_2}\right) - 8 \text{Li}_2\left(\frac{z_1}{s_1}\right) - 8 \text{Li}_2\left(\frac{z_1}{s_2}\right) + 8 \text{Li}_2\left(\frac{z_1}{z_2}\right) \Big], \end{aligned} \quad (24)$$

$$k^{1/2} I_2^{(1)} = -2s_1^2 - 2s_2^2 + 2s_3^2 - 8s_1 s_2 + s_1 s_2 \Big[$$

$$\begin{aligned}
& -32 \log^2(v_1) - 32 \log(v_3) \log(v_1) - 32 \log(v_4) \log(v_1) + 4 \log(v_1) \\
& -8 \log^2(v_3) - 8 \log^2(v_4) - 4 \log(v_2) - 4 \log(v_3) - 16 \log(v_3) \log(v_4) \\
& -16 \text{Li}_2\left(\frac{x_2}{x_1}\right) - 16 \text{Li}_2\left(\frac{y_1}{y_2}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_1}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_2}\right) + 16 \text{Li}_2\left(\frac{z_1}{z_2}\right), \quad (25)
\end{aligned}$$

$$\begin{aligned}
k^{1/2} I_3^{(1)} = & -2s_1^2 + 2s_2^2 + 2s_3^2 - 8s_2s_3 + s_2s_3 \left[ \right. \\
& -32 \log^2(v_1) - 32 \log(v_3) \log(v_1) - 32 \log(v_4) \log(v_1) + 4 \log(v_1) \\
& -8 \log^2(v_3) - 8 \log^2(v_4) - 4 \log(v_2) - 4 \log(v_3) - 16 \log(v_3) \log(v_4) \\
& \left. -16 \text{Li}_2\left(\frac{x_2}{x_1}\right) - 16 \text{Li}_2\left(\frac{y_1}{y_2}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_1}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_2}\right) + 16 \text{Li}_2\left(\frac{z_1}{z_2}\right) \right], \quad (26)
\end{aligned}$$

$$\begin{aligned}
k^{1/2} I_4^{(1)} = & 2s_1^2 - 2s_2^2 + 2s_3^2 - 8s_1s_2 + s_1s_3 \left[ \right. \\
& -32 \log^2(v_1) - 32 \log(v_3) \log(v_1) - 32 \log(v_4) \log(v_1) + 4 \log(v_1) \\
& -8 \log^2(v_3) - 8 \log^2(v_4) - 4 \log(v_2) - 4 \log(v_3) - 16 \log(v_3) \log(v_4) \\
& \left. -16 \text{Li}_2\left(\frac{x_2}{x_1}\right) - 16 \text{Li}_2\left(\frac{y_1}{y_2}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_1}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_2}\right) + 16 \text{Li}_2\left(\frac{z_1}{z_2}\right) \right], \quad (27)
\end{aligned}$$

where the following notation was introduced:

$$x_1 = \frac{1}{2}(s_1 - s_2 - Q^2) - \frac{1}{2}\sqrt{k}, \quad (28)$$

$$x_2 = \frac{1}{2}(s_1 - s_2 - Q^2) + \frac{1}{2}\sqrt{k}, \quad (29)$$

$$y_1 = \frac{1}{2}(s_1 + Q^2 - s_2) - \frac{1}{2}\sqrt{k}, \quad (30)$$

$$y_2 = \frac{1}{2}(s_1 + Q^2 - s_2) + \frac{1}{2}\sqrt{k}, \quad (31)$$

$$z_1 = \frac{1}{2}(s_1 + s_2 + Q^2) - \frac{1}{2}\sqrt{k}, \quad (32)$$

$$z_2 = \frac{1}{2}(s_1 + s_2 + Q^2) + \frac{1}{2}\sqrt{k}, \quad (33)$$

$$v_1 = \frac{1}{2s_1}(s_1 - s_2 - Q^2) + \frac{1}{2s_1}\sqrt{k}, \quad (34)$$

$$v_2 = \frac{1}{2s_2}(s_1 - s_2 + Q^2) + \frac{1}{2s_2}\sqrt{k}, \quad (35)$$

$$v_3 = \frac{1}{2s_1}(s_1 + s_2 + Q^2) + \frac{1}{2s_1}\sqrt{k}, \quad (36)$$

$$v_4 = \frac{s_1}{Q^2}, \quad (37)$$

$$v_5 = \frac{s_2}{Q^2}, \quad (38)$$

$$v_6 = 1 - \frac{z_1}{z_2}. \quad (39)$$

We checked, that all infrared and ultraviolet divergences cancel as should be for vector interpolating currents.

Now, let us proceed with the physical part of three-point sum rules. The connection to hadrons in the framework of QCD sum rules is obtained by matching the resulting QCD expressions of current

correlators with spectral representation, derived from a double dispersion relation at  $q^2 \leq 0$ :

$$\Pi_{\mu\alpha\beta}(p_1^2, p_2^2, q^2) = \frac{1}{(2\pi)^2} \int \frac{\rho_{\mu\alpha\beta}^{\text{phys}}(s_1, s_2, Q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} ds_1 ds_2 + \text{subtractions.} \quad (40)$$

Assuming that the dispersion relation (40) is well convergent, the physical spectral functions are generally saturated by the lowest lying hadronic states plus a continuum starting at some thresholds  $s_1^{th}$  and  $s_2^{th}$ :

$$\begin{aligned} \rho_{\mu\alpha\beta}^{\text{phys}}(s_1, s_2, Q^2) &= \rho_{\mu\alpha\beta}^{\text{res}}(s_1, s_2, Q^2) + \\ &\quad \theta(s_1 - s_1^{th}) \cdot \theta(s_2 - s_2^{th}) \cdot \rho_{\mu\alpha\beta}^{\text{cont}}(s_1, s_2, Q^2), \end{aligned} \quad (41)$$

where

$$\begin{aligned} \rho_{\mu\alpha\beta}^{\text{res}}(s_1, s_2, Q^2) &= \frac{m_\rho^4}{g_\rho^2} \left\{ \right. \\ &\quad \frac{1}{2m_\rho^2} P_\mu(p_{2\alpha}p_{2\beta} + p_{1\alpha}p_{1\beta}) \left[ F_1(Q^2) - F_2(Q^2) + 2 \left( 1 + \frac{Q^2}{2m_\rho^2} \right) F_3(Q^2) \right] \\ &\quad + p_{2\beta}p_{1\alpha}P_\mu \frac{1}{m_\rho^2} \left( 1 + \frac{Q^2}{2m_\rho^2} \right) \left[ F_2(Q^2) - \left( 1 + \frac{Q^2}{2m_\rho^2} \right) F_3(Q^2) \right] \\ &\quad - \frac{1}{m_\rho^2} p_{2\alpha}p_{1\beta}P_\mu F_3(Q^2) + \frac{1}{2m_\rho^2} q_\mu(p_{1\alpha}p_{1\beta} - p_{2\alpha}p_{2\beta}) [F_1(Q^2) + F_2(Q^2)] \\ &\quad - g_{\alpha\beta}P_\mu F_1(Q^2) + (g_{\alpha\mu}p_{1\beta} + g_{\beta\mu}p_{2\alpha}) [F_1(Q^2) + F_2(Q^2)] \\ &\quad - (g_{\alpha\mu}p_{2\beta} + g_{\beta\mu}p_{1\alpha}) \left( 1 + \frac{Q^2}{2m_\rho^2} \right) [F_1(Q^2) + F_2(Q^2)] \left. \right\} (2\pi)^2 \delta(s_1) \delta(s_2) \\ &\quad + \text{higher state contributions} \end{aligned} \quad (42)$$

The continuum of higher states is modeled by the perturbative absorptive part of  $\Pi_{\mu\alpha\beta}$ , i.e. by  $\rho_{\mu\alpha\beta}$ . Then, the expressions for the form factors  $F_i$  can be derived by equating the representations for three-point functions  $\Pi_{\mu\alpha\beta}$  from (7) and (40). It is reasonable to consider 3 sum rules: for structures  $p_{1\mu}g_{\alpha\beta} + p_{2\mu}g_{\alpha\beta}$ ,  $p_{1\mu}p_{2\alpha}p_{1\beta} + p_{2\mu}p_{2\alpha}p_{1\beta}$  and  $p_{1\beta}g_{\alpha\mu} + p_{2\alpha}g_{\beta\mu}$ . This last step constitutes a formulation of QCD sum rules for our particular problem.

### 3 Numerical analysis

For numerical analysis we used Borel scheme of QCD sum rules. That is, to get rid of unknown subtraction terms in (7) we perform Borel transformation procedure in two variables  $s_1$  and  $s_2$ . The Borel transform of three-point function  $\Pi_{\mu\alpha\beta}(s_1, s_2, q^2)$  is defined as

$$\begin{aligned} \Phi_{\mu\alpha\beta}(M_1^2, M_2^2, q^2) &\equiv \hat{B}_{12}\Pi_{\mu\alpha\beta}(s_1, s_2, q^2) = \\ &\quad \lim_{n,m \rightarrow \infty} \left\{ \frac{s_2^{n+1}}{n!} \left( -\frac{d}{ds_2} \right)^n \frac{s_1^{m+1}}{m!} \left( -\frac{d}{ds_1} \right) \Big|_{s_1=mM_1^2, s_2=nM_2^2} \right\} \Pi_{\mu\alpha\beta}(s_1, s_2, q^2) \end{aligned} \quad (43)$$

Then Borel transformation (43) of (7) and (40) gives

$$\Phi_{\mu\alpha\beta}^{(\text{pert}|\text{phys})}(M_1^2, M_2^2, q^2) = \frac{1}{(2\pi)^2} \int_0^\infty ds_1 \int_0^\infty ds_2 \exp \left[ -\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2} \right] \rho_{\mu\alpha\beta}^{(\text{pert}|\text{phys})}(s_1, s_2, q^2), \quad (44)$$

In what follows we put  $M_1^2 = M_2^2 = M^2$ . If  $M^2$  is chosen to be of order  $1 \text{ GeV}^2$ , then the right hand side of (44) in the case of physical spectral density will be dominated by the lowest hadronic state contribution, while the higher state contribution will be suppressed.

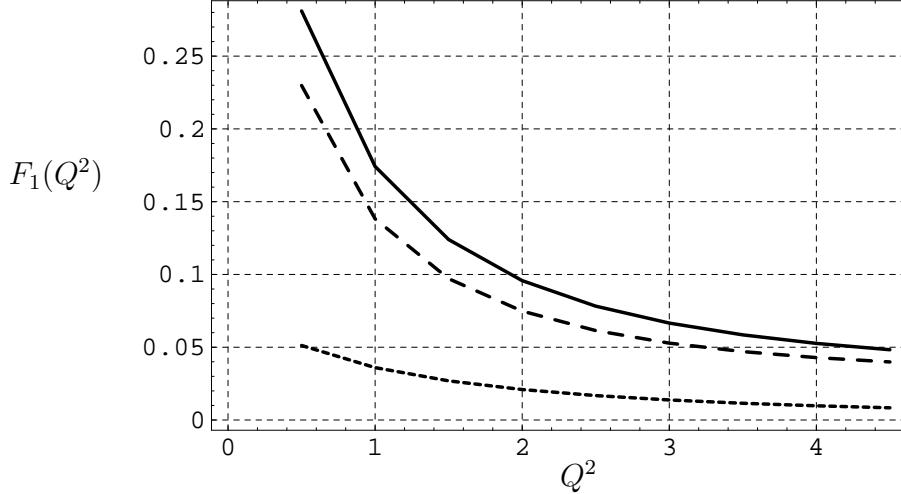


Figure 3:  $Q^2$  dependence of  $F_1$  electric form factor

Now let us recall that our sum rules also receive power corrections proportional to QCD vacuum condensates. The evaluation of power corrections is simplified if performed directly for the Borel transformed expression of three-point correlation function and this is the reason why we delayed their discussion up to his moment. The quark condensate contribution is known already for a long time and is given by [2]:

$$\begin{aligned} \Phi_{\mu\alpha\beta}^{\langle\bar{q}q\rangle^2}(M^2, M^2, q^2) &= \frac{4\pi\alpha_s}{81}\langle 0|\bar{q}q|0\rangle^2 \frac{1}{M^4} \Bigg\{ \\ &\quad P_\mu(p_{1\alpha}p_{1\beta} + p_{2\alpha}p_{2\beta}) \left( 13 - 18\frac{M^2}{Q^2} + 18\frac{M^4}{Q^4} + 2\frac{Q^2}{M^2} \right) \\ &\quad + P_\mu p_{2\beta} p_{1\alpha} \left( 10 + 36\frac{M^2}{Q^2} - 36\frac{M^4}{Q^4} + 4\frac{Q^2}{M^2} \right) + 16P_\mu p_{1\beta} p_{2\alpha} \\ &\quad + q_\mu(p_{1\alpha}p_{1\beta} - p_{2\alpha}p_{2\beta}) \left( 5 - 18\frac{M^2}{Q^2} + 18\frac{M^4}{Q^4} + 2\frac{Q^2}{M^2} \right) \\ &\quad + (p_{2\beta}g_{\alpha\mu} + p_{1\alpha}g_{\beta\mu})M^2 \left( 64 + 5\frac{Q^2}{M^2} - 2\frac{Q^4}{M^4} \right) \\ &\quad + 52P_\mu g_{\alpha\beta}M^2 + (p_{1\beta}g_{\alpha\mu} + p_{2\alpha}g_{\beta\mu})M^2 \left( -20 - 8\frac{Q^2}{M^2} \right) \Bigg\}. \end{aligned} \quad (45)$$

The correction due to gluon condensate was only partially computed in [2], where subset of diagrams as well as part of Lorentz structures were considered. Taking everything into account we get<sup>2</sup>:

$$\Phi_{\mu\alpha\beta}^{\langle G^2 \rangle}(M^2, M^2, q^2) = \frac{\alpha_s}{48\pi}\langle 0|G_{\rho\sigma}^a G_{\rho\sigma}^a|0\rangle \Bigg\{$$

<sup>2</sup>See Appendix A for the details of calculation

$$\begin{aligned}
& P_\mu p_{1\beta} p_{2\alpha} \frac{4}{Q^2} + P_\mu p_{1\alpha} p_{2\beta} \frac{2}{M^2} + (p_{1\mu} p_{2\beta} p_{2\alpha} + p_{2\mu} p_{1\beta} p_{1\alpha}) \left( -\frac{4}{Q^2} + \frac{2}{M^2} \right) \\
& - 2g_{\mu\beta} p_{2\alpha} - 2p_{1\beta} g_{\mu\alpha} - \frac{Q^2}{M^2} p_{2\beta} g_{\mu\alpha} - \frac{Q^2}{M^2} p_{1\alpha} g_{\mu\beta} + 5p_{2\beta} g_{\mu\alpha} + 5p_{1\alpha} g_{\mu\beta} \Big\}. \quad (46)
\end{aligned}$$

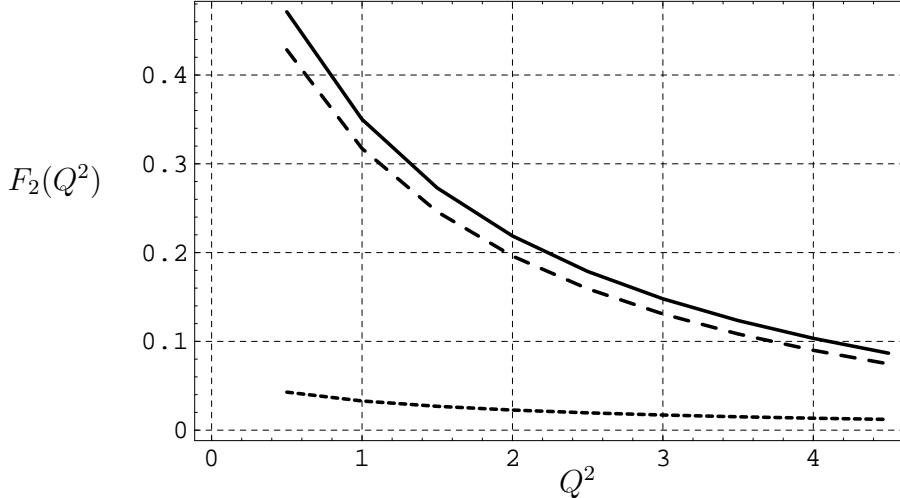


Figure 4:  $Q^2$  dependence of  $F_2$  magnetic form factor

Equating Borel transformed theoretical and physical parts of QCD sum rules we get

$$\begin{aligned}
F_1(Q^2) &= \frac{g_\rho^2}{8\pi^2} \left( \frac{M^2}{m_\rho^2} \right)^2 \exp \left[ \frac{2m_\rho^2}{M^2} \right] \Bigg\{ \\
&\int_0^{\frac{s_0}{M^2}} dx' \int_0^{x'} dy' s'_3(x' + s'_3) A_2^{(0)} \exp[-x'] \\
&+ \frac{\alpha_s}{4\pi} \int_0^{\frac{s_0}{M^2}} dx' \int_0^{x'} dy' s'_3(x' + s'_3) A_2^{(1)} \exp[-x'] + \frac{640\pi^3}{81M^6} \alpha_s \langle 0 | \bar{q}q | 0 \rangle^2 \Bigg\}, \quad (47)
\end{aligned}$$

$$\begin{aligned}
F_2(Q^2) &= \frac{g_\rho^2}{8\pi^2} \left( \frac{M^2}{m_\rho^2} \right)^2 \exp \left[ \frac{2m_\rho^2}{M^2} \right] \Bigg\{ \\
&\int_0^{\frac{s_0}{M^2}} dx' \int_0^{x'} dy' \left[ s'_1(y' + s'_3) A_3^{(0)} - s'_2(y' - s'_3) A_4^{(0)} - s'_3(x' + s'_3) A_2^{(0)} \right] \exp[-x'] \\
&+ \frac{\alpha_s}{4\pi} \int_0^{\frac{s_0}{M^2}} dx' \int_0^{x'} dy' \left[ s'_1(y' + s'_3) A_3^{(1)} - s'_2(y' - s'_3) A_4^{(1)} - s'_3(x' + s'_3) A_2^{(1)} \right] \exp[-x'] \\
&+ \frac{256\pi^3}{81M^6} \alpha_s \langle 0 | \bar{q}q | 0 \rangle^2 (4 - s'_3) - \frac{\pi^2}{3M^4} \langle 0 | \frac{\alpha_s}{\pi} G_{\rho\sigma}^2 | 0 \rangle \Bigg\}, \quad (48)
\end{aligned}$$

$$\begin{aligned}
F_3(Q^2) &= \frac{g_\rho^2}{8\pi^2} \left( \frac{M^2}{m_\rho^2} \right) \exp \left[ \frac{2m_\rho^2}{M^2} \right] \Bigg\{ \\
&\int_0^{\frac{s_0}{M^2}} dx' \int_0^{x'} dy' \left[ 2s'_1 A_3^{(0)} + 2s'_2 A_4^{(0)} - 2s'_3 A_2^{(0)} - 8s'_1 s'_2 s'_3 A_1^{(0)} \right] \exp[-x'] \quad (49)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\alpha_s}{4\pi} \int_0^{s_0/M^2} dx' \int_0^{x'} dy' \left[ 2s'_1 A_3^{(1)} + 2s'_2 A_4^{(1)} - 2s'_3 A_2^{(1)} - 8s'_1 s'_2 s'_3 A_1^{(1)} \right] \exp[-x'] \\
& - \frac{512\pi^3}{81M^6} \alpha_s \langle 0 | \bar{q}q | 0 \rangle^2 - \frac{2\pi^2}{3M^4 s'_3} \langle 0 | \frac{\alpha_s}{\pi} G_{\rho\sigma}^2 | 0 \rangle \Big\}, \tag{49}
\end{aligned}$$

where the following notation  $x' = \frac{x}{M^2}$ ,  $y' = \frac{y}{M^2}$ ,  $s'_1 = \frac{s_1}{M^2}$ ,  $s'_2 = \frac{s_2}{M^2}$  and  $s'_3 = \frac{Q^2}{M^2}$  was introduced.

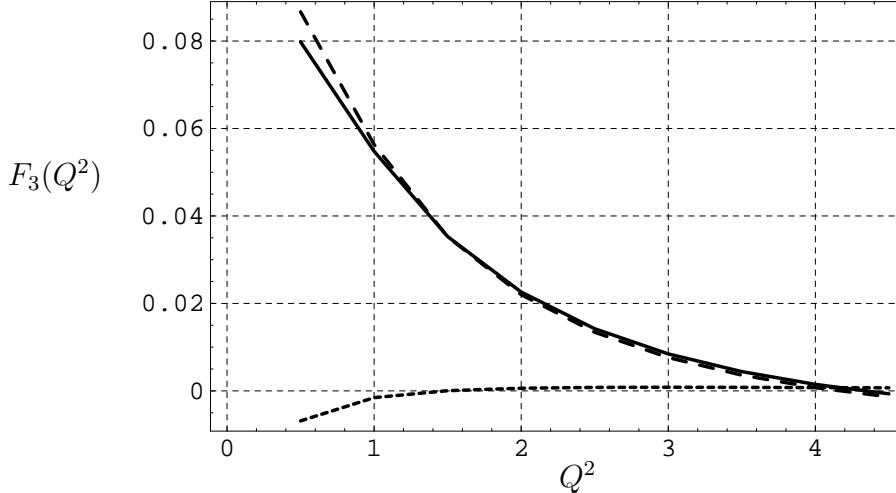


Figure 5:  $Q^2$  dependence of  $F_3$  quadrupole form factor

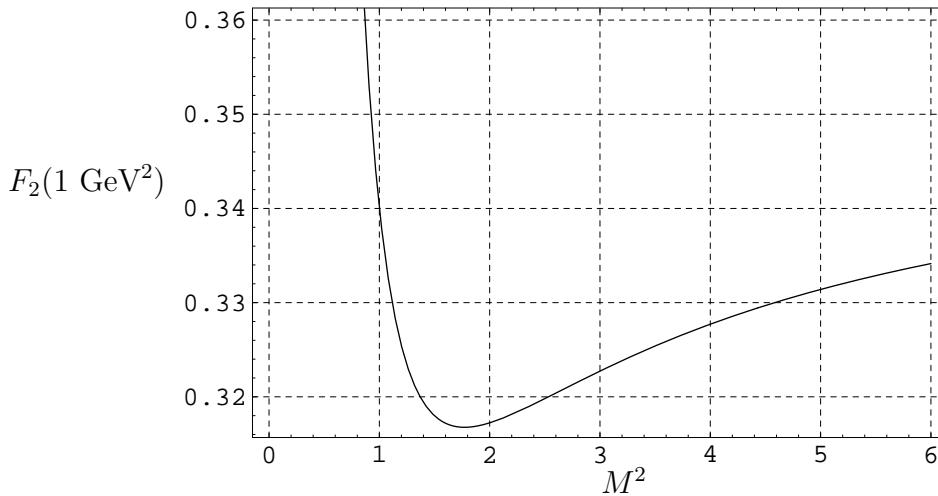


Figure 6: Borel mass  $M^2$  dependence of  $\rho$ -meson magnetic form factor at  $Q^2 = 1 \text{ GeV}^2$

Here, for continuum subtraction we used so called "triangle" model. To verify the stability of our results with respect to the choice of continuum model we checked, that the usual "square" model gives similar predictions for pion electromagnetic form factor provided  $s_0 \sim 1.5s_1$  is chosen<sup>3</sup>. In

<sup>3</sup>For more information about different continuum subtraction models see [2]

what follows we set  $s_0 = 2.2$ . This value coincides with one used in the previous analysis [2] and is in agreement with the value of continuum threshold employed in the analysis of corresponding two-point QCD sum rules. For the rest of parameters entering our expressions for form factors we used the following values<sup>4</sup>:

$$\begin{aligned} m_\rho &= 0.77 \text{ GeV}, \\ \langle 0 | \frac{\alpha_s}{\pi} G_{\rho\sigma}^2 | 0 \rangle &= 0.009 \text{ GeV}^4, \\ \alpha_s \langle 0 | \bar{q}q | 0 \rangle^2 &= 2.2 \cdot 10^{-4} \text{ GeV}^6. \end{aligned}$$

In Fig. 6 we plotted the dependence of the  $\rho$ -meson magnetic form factor  $F_2(Q^2)$  from the value of Borel parameter  $M^2$  at  $Q^2 = 1 \text{ GeV}^2$ . Similar dependence was also found for other form factors. To ensure stability of our estimates to the variation of Borel parameter we fix its value at  $M^2 = 2 \text{ GeV}^2$ , belonging to the "stability plateau".

The results for  $\rho$ -meson electric form factor including NLO corrections are shown in Fig. 3 (solid line is the sum of LO and NLO contributions, curve with long dashes denotes LO contribution and curve with short dashes stands for NLO contributions.). Similar results for magnetic  $F_2(Q^2)$  and quadrupole  $F_3(Q^2)$  form factors are shown in Figs. 4 and 5 correspondingly (again solid line is the sum of LO and NLO contributions, curve with long dashes denotes LO contribution and curve with short dashes stands for NLO contributions.). Unfortunately, our sum rules do not allow us determine magnetic moment of  $\rho$ -meson with precision better than already available predictions both from sum rules [6, 7] and different quark models [8, 9, 10]. At small  $Q^2$  our formula simply do not work - at this point we approach physical region of our three-point correlation function and OPE used in our analysis breaks down. Here one may only conclude, that the value of  $\rho$ -meson magnetic moment is close to 2 by extrapolating the ratio of  $F_2(Q^2)/F_1(Q^2)$  into the region of small momentum transfer  $Q^2$  [2].

Finally, let us following [2] compare the behaviour of our form factors with radiative corrections taken into account in the limit  $Q^2 \rightarrow \infty$  with that, predicted by perturbative QCD (pQCD). The asymptotic of  $\rho$ -meson form factors is typically discussed in terms of states with given transverse  $\epsilon_T$  and longitudinal  $\epsilon_L$  polarizations of  $\rho$ -meson. In Breit system these two descriptions could be related with the help of following formula:

$$F_{TT}(Q^2) = \langle p', \epsilon_T | j_0^{\text{el}} | p, \epsilon_T \rangle / 2E = F_1(Q^2), \quad (50)$$

$$F_{LT}(Q^2) = \langle p', \epsilon_T | j_x^{\text{el}} | p, \epsilon_L \rangle / 2E = \frac{Q}{2m_\rho} (F_1(Q^2) + F_2(Q^2)), \quad (51)$$

$$F_{LL}(Q^2) = \langle p', \epsilon_L | j_0^{\text{el}} | p, \epsilon_L \rangle / 2E = F_1(Q^2) - \frac{Q^2}{2m_\rho^2} F_2(Q^2) + \frac{Q^2}{m_\rho^2} \left( 1 + \frac{Q^2}{4m_\rho^2} \right) F_3(Q^2), \quad (52)$$

where  $E$  is the meson energy  $E = \sqrt{m_\rho^2 + \frac{1}{4}Q^2}$ .

Quark counting and chirality conservation lead to the following asymptotic behavior of form factors:  $F_{LL}(Q^2) \sim 1/Q^2$ ,  $F_{LT}(Q^2) \sim 1/Q^3$  and  $F_{TT}(Q^2) \sim 1/Q^2$ . In terms of electric, magnetic and quadrupole form factors we have  $F_1(Q^2) \sim F_2(Q^2) \sim 1/Q^4$  and  $F_3(Q^2) \sim 1/Q^6$ . It is easy to check that our form factors with radiative corrections included follow this behavior in asymptotic limit, while LO results contribute only as power corrections at large momentum transfers.

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<sup>4</sup>For numerical values of QCD condensates we took central values of estimates made in [5]

## 4 Conclusion

We presented the results for  $\rho$ -meson electromagnetic form factors in the framework of three-point NLO QCD sum rules. The radiative corrections are sizeable ( $\sim 30\%$  in the case of  $F_1$  form factor and somewhat smaller for two other form factors) and should be taken into account when precision data on  $\rho$ -meson form factors become available.

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## A Gluon condensate correction

Here we present details on the evaluation of power corrections proportional to gluon condensate. This calculation could be relatively easily performed directly for the Borel transformed expression of three-point correlation function. Unfortunately, one of the methods (calculation in coordinate space), we will discuss below, does not allow us to subtract continuum contribution for gluon condensate correction. However, the form of the obtained expression leads us to the conclusion, that this contribution is simply absent in our final result. This conclusion is based on a fact, that typical

continuum contribution may show up only as incomplete  $\Gamma$ -functions. The latter are in fact present in contributions of each separate diagram, but they are canceling in the sum.

The gluon condensate contribution to the three-point sum rules is given by diagrams with two external gluon vacuum fields, depicted in Fig. 7. For calculations we have used the Fock-Schwinger fixed point gauge [11, 12]:

$$x^\mu A_\mu^a(x) = 0, \quad (53)$$

where  $A_\mu^a$ ,  $a = \{1, 2, \dots, 8\}$  is the gluon field. The use of this gauge allows us express gauge potential  $A_\mu(x)$  in terms of field strength and its covariant derivatives at origin:

$$A_\mu^a(x) = -\frac{1}{2}x_\nu G_{\mu\nu}^a(0) - \frac{1}{3}x_\nu x_\alpha (D_\alpha G_{\mu\nu})^a(0) + \dots \quad (54)$$

or in momentum representation:

$$\begin{aligned} A_\mu^a(k) &= -\frac{1}{2}i(2\pi)^4 G_{\nu\mu}(0) \frac{\partial}{\partial k_\nu} \delta^4(k) \\ &\quad - \frac{1}{3}(2\pi)^4 (D_\alpha G_{\nu\mu}(0))^a \frac{\partial^2}{\partial k_\alpha \partial k_\nu} \delta^4(k) + \dots \end{aligned} \quad (55)$$

So, basically, the calculation of gluon condensate correction is the ordinary calculation in background of vacuum gluon fields in the form of (54) or (55). Finally, vacuum averaging is performed according to rule:

$$\langle 0 | G_{\mu\nu} G_{\rho\sigma}^b | 0 \rangle = \frac{1}{96} \delta^{ab} \langle 0 | G_{\alpha\beta}^c G_{\alpha\beta}^c | 0 \rangle (\delta_{\mu\rho} \delta_{\nu\sigma} - \delta_{\mu\sigma} \delta_{\nu\rho}) \quad (56)$$

The calculation could be simplified if one uses the expression for quark propagator in background of gluon vacuum field:

$$\begin{aligned} S(x, y) &= \frac{1}{2\pi^2} \frac{r'}{(r^2)^2} - \frac{1}{8\pi^2} \frac{r_\alpha}{r^2} \tilde{G}_{\alpha\beta}(0) \gamma_\beta \gamma_5 \\ &\quad + \left\{ \frac{i}{4\pi^2} \frac{r'}{(r^2)^2} y_\alpha x_\beta G_{\alpha\beta}(0) - \frac{1}{192\pi^2} \frac{r'}{(r^2)^2} (x^2 y^2 - (xy)^2) G_{\alpha\beta}(0) G_{\alpha\beta}(0) \right\} \\ &\quad + \text{operators of higher dimension,} \end{aligned} \quad (57)$$

where

$$r = x - y, \quad G_{\alpha\beta} = \frac{g}{2} \lambda^a G_{\alpha\beta}^a, \quad \tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\mu\nu} G_{\mu\nu} \quad (58)$$

The corresponding momentum representation of quark propagator could be easily obtained through Fourier transformation. Next, for coordinates chosen as indicated in Fig. 7 from (57) it follows that diagrams *a*) and *d*) are identically zero.

Now there are two ways to proceed: first is to perform the whole calculation in momentum representation [2] and second way - do the same in coordinate space. Let's consider first calculation in momentum space. It is easy to find that the contribution of diagrams, where external gluon fields attached to different quark lines, is given by

$$\Pi_{\mu\alpha\beta}^{\mathbf{b}+\mathbf{e}+\mathbf{f}}(p_1, p_2, q) = i \frac{g^2 \langle 0 | G_{\rho\sigma}^2 | 0 \rangle}{24 (2\pi)^4} \int d^4 k \frac{1}{k^2 P_1^2 P_2^2} \left\{ \dots \right\}$$

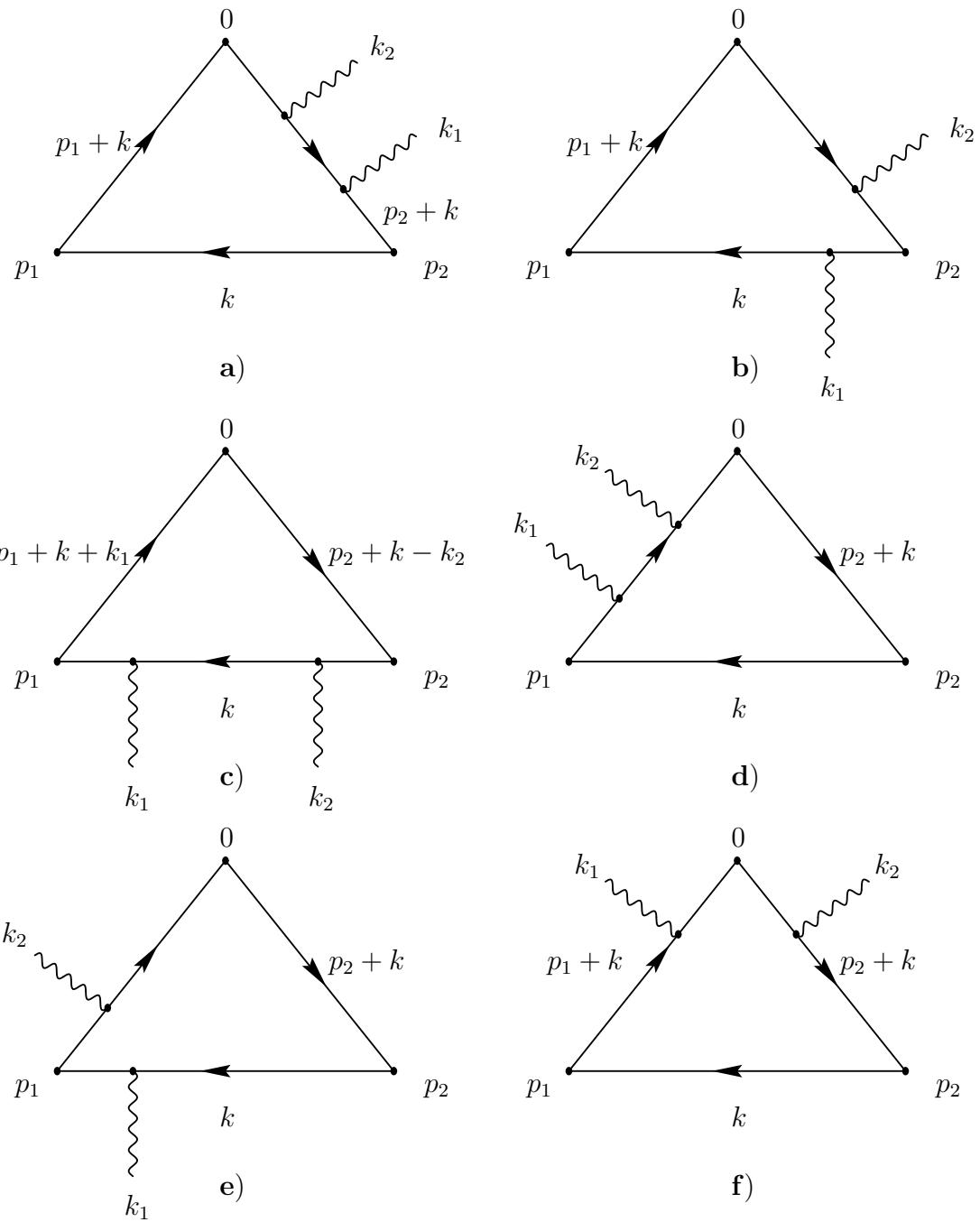


Figure 7: The gluon condensate contribution to three-point QCD sum rules. The directions of  $p_1$ ,  $k_1$ ,  $k_2$  momenta are incoming, and that of  $p_2$  is outgoing.

$$\begin{aligned}
& + \frac{1}{k^2 P_1^2} \text{Tr}[\gamma^\mu \hat{k} \gamma^\alpha \hat{P}_1 \gamma^\beta \hat{P}_2 + 2\gamma^\mu \gamma^\alpha \gamma^\beta \hat{P}_2 P_1 \cdot k] \\
& + \frac{1}{P_1^2 P_2^2} \text{Tr}[\gamma^\mu \hat{P}_2 \gamma^\alpha \hat{k} \gamma^\beta \hat{P}_1 + 2\gamma^\mu \gamma^\alpha \gamma^\beta \hat{k} \gamma^\alpha P_1 \cdot P_2] \\
& + \frac{1}{k^2 P_2^2} \text{Tr}[\gamma^\mu \hat{P}_1 \gamma^\alpha \hat{P}_2 \gamma^\beta \hat{k} + 2\gamma^\mu \hat{P}_1 \gamma^\alpha \gamma^\beta k \cdot P_2] \Big\}, \tag{59}
\end{aligned}$$

where  $P_1 = p_1 + k$  and  $P_2 = p_2 + k$ . The integrals entering this expression could be conveniently evaluated using double spectral representation. For example,

$$\begin{aligned}
\int d^4k \frac{k_\mu P_{1\rho} P_{2\sigma}}{k^2 P_1^4 P_2^4} &= \frac{1}{4} \frac{\partial^2}{\partial p_{1\rho} \partial p_{2\sigma}} \int d^4k \frac{k_\mu}{k^2 P_1^2 P_2^2} \\
&= -\frac{i}{4} \frac{\partial^2}{\partial p_{1\rho} \partial p_{2\sigma}} \int \int ds_1 ds_2 \frac{\pi^2}{\lambda^{3/2}} \frac{s_2(s_1 - s_2 - Q^2)p_{1\mu} + s_1(s_2 - s_1 - Q^2)p_{2\mu}}{(s_1 - p_1^2)(s_2 - p_2^2)} \tag{60}
\end{aligned}$$

and similar expressions hold for other two integrals. Finally, for the remained diagram **c**) the use of quark propagator in the background vacuum gluon field (57) allows present this contribution in the following compact form:

$$\Pi_{\mu\alpha\beta}^c(p_1, p_2, q) = \frac{g^2}{576} \langle 0 | G_{\mu\nu}^2 | 0 \rangle \left[ \frac{\partial^4}{\partial p_{1\rho} \partial p_{2\rho} \partial p_{1\sigma} \partial p_{2\sigma}} - \frac{\partial^2}{\partial p_{1\rho} \partial p_{1\rho}} \frac{\partial^2}{\partial p_{2\sigma} \partial p_{2\sigma}} \right] \Pi_{\mu\alpha\beta}^{(0)}(p_1, p_2, q), \tag{61}$$

where  $\Pi_{\mu\alpha\beta}^{(0)}(p_1, p_2, q)$  is LO perturbative contribution to our three-point correlation function. Performing all differentiations and doing afterwards Borel transform according to

$$\widehat{B}_{P^2}(M^2) \frac{1}{(P^2 - m^2)^n} = \frac{1}{(n-1)!} (-1)^n \frac{1}{(M^2)^n} e^{-m^2/M^2} \tag{62}$$

we come to the final expression for gluon condensate contribution presented in the main body of the paper.

Now, let us make a few comments about calculation of gluon condensate contribution within coordinate space representation<sup>5</sup>. The coordinate space amplitude corresponding to this contribution easily follows from an expression of quark propagator in the background gluon field (57). However, its Borel transformation is not that trivial. To do it, we first convert our result into momentum space with the help of the following formula [14]:

$$\frac{1}{\pi^4} \int e^{ip_2x - ip_1y} \frac{d^4x d^4y}{(x-y)^{2l} y^{2m} x^{2n}} = \frac{(-1)^{l+m+n+1}}{4^{l+n+m+1} l! m! n!} \int_0^\infty e^{\tau_1 p_1^2 + \tau_2 p_2^2 + \tau_3 q^2} \frac{d\tau_1^n d\tau_2^m d\tau_3^l}{(\tau_1 \tau_2 + \tau_2 \tau_3 + \tau_3 \tau_1)^{l+m+n-2}} \tag{63}$$

The factors in numerator could be incorporated via:

$$x_\mu \rightarrow -i \frac{\partial}{\partial p_{2\mu}}, \quad y_\mu \rightarrow i \frac{\partial}{\partial p_{1\mu}}. \tag{64}$$

The Borel transformation of the resulting expression is performed with the help of the following formula:

$$\widehat{B}_{P^2}(M^2) e^{-\tau P^2} = \delta(1 - \tau M^2) \tag{65}$$

The final expression for gluon condensate contribution obtained within this approach coincides with the result obtained in momentum representation and serves as a check of our result.

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<sup>5</sup>For more information see also appendix in [13]